SAFE HANDS & IIT-ian's PACE

LEAP TEST# 07 (JEE) ANS KEY Dt. 17-12-2023

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	D	31	D	61	D
2	A	32	C	62	A
3	D	33	B	63	A
4	A	34	D	64	B
5	D	35	A	65	B
6	C	36	D	66	A
7	C	37	B	67	A
8	B	38	D	68	C
9	A	39	C	69	D
10	A	40	A	70	B
11	A	41	C	71	D
12	A	42	C	72	D
13	A	43	D	73	B
14	D	44	C	74	C
15	C	45	C	75	A
16	B	46	B	76	C
17	D	47	A	77	D
18	A	48	C	78	A
19	A	49	C	79	D
20	C	50	D	80	C
21	2	51	34	81	4
22	2.95	52	1485	82	0
23	66.67	53	117	83	2
24	24.8	54	0.495	84	0
25	2	55	14.5	85	1

See Maths solutions on next page.....

SAFE HANDS & PACE

LT-07 (JEE) Mathematics Solutions

: ANSWER KEY :

61)	d	62)	a	63)	a	64)	b	77)	d	78)	a	79)	d	80)	c
65)	b	66)	a	67)	a	68)	c	81)	4	82)	0	83)	2	84)	0
69)	d	70)	b	71)	d	72)	d	85)	1						
73)	b	74)	c	75)	a	76)	c								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **(d)**

Since, $f(x)=\begin{cases}\frac{\sin[x]}{[x]}, & [x]\neq 0\\ 0, & [x]=0\end{cases}$

$\Rightarrow \quad f(x)=\begin{cases}\frac{\sin[x]}{[x]}, & x\in R-[0,1)\\ 0, & 0\le x<1\end{cases}$

At $x=0$,

$$\text{RHL}=\lim_{x\to 0^+}0=0$$

and $\quad \text{LHL}=\lim_{x\to 0^-}\frac{\sin[x]}{[x]}=\lim_{h\to 0}\frac{\sin[0-h]}{[0-h]}$

$$=\lim_{h\to 0}\frac{\sin(-1)}{-1}=\sin 1$$

Since, RHL $\neq$ LHL

$\therefore$ Limit does not exist.

62 **(a)**

We have,

$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2} \quad \left[\frac{0}{0}\text{ form}\right]$

$=\lim_{x\to 0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2}$ [Using L' Hospital's Rule]

$=\lim_{x\to 0}\frac{2f''(x)12f''(2x)+16f''(4x)}{2}$ [Using L' Hospital's Rule]

$=\frac{2f''(0)-12f''(0)+16f''(0)}{2}$

$=3f''(0)=3\times 2=6$

63 **(a)**

Here, $\lim_{x\to -3}x^2+2x-3=0$

$\therefore \lim_{x\to -3}3x^2+ax+a-7$ must be zero, in order to limit exist.

$\Rightarrow \quad 3(-3)^2+a(-3)+a-7=0$

$\Rightarrow \quad 27-2a-7=0$

$\Rightarrow \quad 2a=20$

$\Rightarrow \quad a=10$

64 **(b)**

$$\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$$

NOTE In trigonometry try to make all trigonometric functions in same angle. It is called 3rd Golden rule of trigonometry.

$$=\lim_{x\to 0}\frac{x\frac{2\tan x}{1-\tan^2 x}-2x\tan x}{(2\sin^2 x)^2}$$

$$=\lim_{x\to 0}\frac{2x\tan x\left[\frac{1}{1-\tan^2 x}-1\right]}{4\sin^4 x}$$

$$=\lim_{x\to 0}\frac{2x\tan x\left[\frac{1-1+\tan^2 x}{1-\tan^2 x}\right]}{4\sin^4 x}$$

$$=\lim_{x\to 0}\frac{2x\tan^3 x}{2\sin^4 x(1-\tan^2 x)}$$

$$=\lim_{x\to 0}\frac{1}{2}\frac{x\left(\frac{\tan x}{x}\right)^3\cdot x^3}{\sin^4 x\,(1-\tan^2 x)}$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{\left(\frac{\tan x}{x}\right)^3}{\left(\frac{\sin x}{x}\right)^4(1-\tan^2 x)}=\frac{1\cdot(1)^3}{2(1)^4\,(1-0)}=\frac{1}{2}$$

65 **(b)**

We have,

$\lim_{x\to 0}x^2\sin\frac{\pi}{x}$

$=0\times$ (A finite oscillating number) $=0$

66 **(a)**

$$\left(\lim_{x\to\infty}\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right.$$

$$\left.\times\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\right)$$

$$=\lim_{y\to 0}\frac{(\sqrt{1+\sqrt{y}})/\sqrt{y}}{\sqrt{\frac{1}{y}+\frac{\sqrt{1+\sqrt{y}}}{\sqrt{y}}}+\sqrt{\frac{1}{y}}} \qquad \left[\text{put } x=\frac{1}{y}\right]$$

$$=\lim_{y\to 0}\frac{\sqrt{1+\sqrt{y}}}{\sqrt{1+\sqrt{y(1+\sqrt{y})}}+1}=\frac{1}{2}$$

67 **(a)**

$$\lim_{x\to 0}\frac{a^x-b^x}{e^x-1}=\lim_{x\to 0}\frac{a^x-b^x}{x}\cdot\frac{x}{a^x-1}$$

$$=\left[\lim_{x\to 0}\left(\frac{a^x-1}{x}\right)-\lim_{x\to 0}\left(\frac{b^x-1}{x}\right)\right].\lim_{x\to 0}\frac{x}{e^x-1}$$

$$= (\log_e a - \log_e b).\lim_{x\to 0}\frac{1}{\frac{e^x-1}{x}}$$

$$= \log_e\left(\frac{a}{b}\right)$$

68 **(c)**

We have,

$$\lim_{x\to\infty}\frac{5^{x+1}-7^{x+1}}{5^x-7^x} = \lim_{x\to\infty}\frac{5.\left(\frac{5}{7}\right)^x-7}{\left(\frac{5}{7}\right)^x-1} = \frac{5\times 0-7}{0-1}$$

$$= 7$$

69 **(d)**

We have,

$$\lim_{x\to 3}\frac{f(x)-f(3)}{x-3} = f'(3)$$

Now, $f'(x) = \frac{x}{(18-x^2)^{3/2}} \Rightarrow f'(3) = \frac{3}{(9)^{3/2}} = \frac{1}{9}$

70 **(b)**

We have,

$$\lim_{x\to 1}(\log_4 5x)^{\log_x 5} = \lim_{x\to 1}(\log_5 5+\log_5 x)^{\log_x 5}$$

$$= \lim_{x\to 1}(1+\log_5 x)^{\frac{1}{\log_5 x}} = e^{\lim_{x\to 1}\log_5 x.\frac{1}{\log_x 5}} = e^1 = e$$

71 **(d)**

$$\lim_{x\to 0}\frac{(1-\cos 2x)}{x^2}$$

$$= \lim_{x\to 0}\frac{2\sin^2 x}{x^2}$$

$$= 2\lim_{x\to 0}\left(\frac{\sin x}{x^2}\right)^2 = 2$$

72 **(d)**

We have,

$$\lim_{x\to\infty}\left(\frac{x^2+2x+3}{2x^2+x+5}\right)^{\frac{3x-2}{3x+2}} = \lim_{x\to\infty}\left(\frac{1+\frac{2}{x}+\frac{3}{x^2}}{2+\frac{1}{x}+\frac{5}{x^2}}\right)^{\frac{1-2/3x}{1+2/3x}}$$

$$= \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$

73 **(b)**

$$\lim_{x\to 0}x^2\sin\left(\frac{\pi}{x}\right) = \lim_{x\to 0}\pi x.\frac{\sin\frac{\pi}{x}}{\frac{\pi}{x}} = 0(1) = 0$$

74 **(c)**

We have,

$$\lim_{x\to\infty}x\left\{\tan^{-1}\frac{x+1}{x+2}-\tan^{-1}\frac{x}{x+2}\right\}$$

$$= \lim_{x\to\infty}x\tan^{-1}\left\{\frac{\frac{x+1}{x+2}-\frac{x}{x+2}}{1+\frac{x+1}{x+2}.\frac{x}{x+2}}\right\}$$

$$= \lim_{x\to\infty}x\tan^{-1}\left(\frac{x+2}{2x^2+5x+4}\right)$$

$$= \lim_{x\to\infty}\left\{\frac{\tan^{-1}\left(\frac{x+2}{2x^2+5x+4}\right)}{\frac{x+2}{2x^2+5x+4}}\right\}\times\frac{x(x+2)}{2x^2+5x+4}$$

$$= 1\times\frac{1}{2} = \frac{1}{2}$$

75 **(a)**

We have,

$$\lim_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x-x} = \lim_{x\to 0}\frac{e^x\{e^{\tan x-x}-1\}}{\tan x-x}$$

$$= \lim_{x\to 0}e^x\times\lim_{x\to 0}\frac{e^{\tan x-x}-1}{\tan x-x}e^0\times 1 = 1$$

76 **(c)**

$$\lim_{h\to 0}\frac{\sin\sqrt{x+h}-\sin\sqrt{x}}{h}$$

Applying L'Hospital's rule,

$$= \lim_{h\to 0}\frac{\frac{\cos\sqrt{x+h}}{2\sqrt{x+h}}}{1} = \frac{\cos\sqrt{x}}{2\sqrt{x}}$$

77 **(d)**

We have,

$A_i = \frac{x-a_i}{|x-a_i|}, i = 1,2,\dots,n$ and $a_1 < a_2 < \dots <$ $a_{n-1} < a_n$

Let x be in the left neighbourhood of a_m. Then,

$x-a_i < 0$ for $i = m, m+1, \dots, n$

and,

$x-a_i > 0$ for $i = 1,2,\dots,m-1$

$$A_i = \begin{cases} = \frac{(x-a_i)}{-(x-a_i)} = -1 \text{ for } i = m, m+1,\dots,n \\ = \frac{x-a_i}{x-a_i} = 1 \text{ for } i = 1,2,\dots,m-1 \end{cases}$$

Similarly, if x is in the right neighbourhood of a_m. Then,

$x-a_i < 0$ for $i = m+1,\dots,n$ and $x-a_i > 0$ for $i = 1,2,\dots,m$

$$\therefore A_i = \begin{cases} A_i = \frac{x-a_i}{-(x-a_i)} = -1 \text{ for } i = m+1,\dots,n \\ A_i = \frac{x-a_i}{x-a_i} = 1 \text{ for } i = 1,2,\dots,m \end{cases}$$

Thus, we have

$$\lim_{x\to a_m^-}(A_1 A_2 \dots A_n) = (-1)^{n-m+1}$$

and,

$$\lim_{x\to a_m^+}(A_1 A_2 \dots A_n) = (-1)^{n-m}$$

Hence, $\lim_{x\to a_m}(A_1 A_2 \dots A_n)$ does not exist

78 **(a)**

We have,

$$\lim_{x\to -1}\frac{(1+x)(1-x^2)(1+x^3)(1-x^4)\dots(1+x^{4n-}}{[(1+x)(1-x^2)(1+x^3)(1-x^4)\dots(1-}$$

$$= \lim_{x\to -1}\frac{(1+x^{2n+1})(1-x^{2n+2})\dots(1+x^{4n-1})(1-}{(1+x)(1-x^2)(1+x^3)(1-x^4)\dots(1-x}$$

$$= \lim_{x\to -1}\left\{\frac{1+x^{2n+1}}{1+x}\times\frac{1-x^{2n+2}}{1-x^2}\times\frac{1+x^{2n+3}}{1+x^3}\times\dots\times\frac{1-x^{4n}}{1-x^{2n}}\right\}$$

$$= \lim_{x\to -1}\left\{\frac{x^{2n+1}+1}{x+1}\times\frac{x^{2n+2}-1}{x^2-1}\times\frac{x^{2n+3}+1}{x^3+1}\times\dots\times\frac{x^{4n}-1}{x^{2n}-1}\right\}$$

$$= \lim_{x\to -1}\left\{\frac{x^{2n+1}-(-1)^{2n+1}}{x-(-1)}\times\frac{x^{2n+2}-(-1)^{2n+2}}{x^2-(-1)^2}\right\}\times\left\{\frac{x^{2n+3}-(-1)^{2n+3}}{x^3-(-1)^3}\times\dots\times\frac{x^{4n}-(-1)^{4n}}{x^{2n}-(-1)^{2n}}\right\}$$

$$= \frac{2n+1}{1}\times\frac{2n+2}{2}\times\frac{2n+3}{3}\times\dots\times\frac{4n}{2n}$$

$$= \frac{4n!}{\{(2n)!\}^2} = {}^{4n}C_{2n}$$

79 **(d)**

$\lim_{x\to a}\frac{f(a)g(x)-f(x)g(a)}{x-a}$ $\left[\frac{0}{0}\text{ from}\right]$

$= \lim_{x\to a}\frac{f(a)g'(x)-f'(x)g(a)}{1-0}$ [by L' Hospital's rule]

$= f(a)g'(a)-f'(a)g(a)$

$= 2(-1)-1(3) = -2-3 = -5$

80 **(c)**

We have,

$$\lim_{x\to\pi/2}\tan^2 x\left(\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right)$$

$$= \lim_{x\to\pi/2}\tan^2 x\frac{(2\sin^2 x+3\sin x+4-\sin^2 x-6\sin x-2)}{\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2}}$$

$$= \lim_{x\to\pi/2}\frac{\tan^2 x(\sin^2 x-3\sin x+2)}{\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2}}$$

$$= \lim_{x\to\pi/2}\frac{\sin^2 x(\sin x-1)(\sin x-2)}{(1-\sin^2 x)(\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2})}$$

$$= \lim_{x\to\pi/2}\frac{-\sin^2 x(\sin x-2)}{(1+\sin x)(\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2})}$$

$$= \frac{1}{2(\sqrt{9}+\sqrt{9})} = \frac{1}{12}$$

Integer Answer Type

81 **(4)**

Let $x = 1/y$

$$\Rightarrow \lim_{x\to\infty}\left(x-x^2\log_e\left(1+\frac{1}{x}\right)\right)$$

$$= \lim_{y\to 0}\left(\frac{1}{y}-\frac{\log_e(1+y)}{y^2}\right)$$

$$= \lim_{y\to 0}\left(\frac{y-\log_e(1+y)}{y^2}\right)$$

$$= \lim_{y\to 0}\left(\frac{y-\left(y-\frac{y^2}{2}\right)}{y^2}\right) = 1/2$$

82 **(0)**

$$\mathrm{f(x)} = \tan^{-1}\left(\frac{\log(\mathrm{ex}^{-2})}{\log \mathrm{ex}^2}\right)+\tan^{-1}\left(\frac{3+2\log \mathrm{x}}{1-6\log \mathrm{x}}\right)$$

$$= \tan^{-1}\left(\frac{\log \mathrm{e}+\log \mathrm{x}^{-2}}{\log \mathrm{e}+\log \mathrm{x}^2}\right)+\tan^{-1}\left(\frac{3+2\log \mathrm{x}}{1-3(2\log \mathrm{x})}\right)$$

$$= \tan^{-1}\left(\frac{1-2\log \mathrm{x}}{1+2\log \mathrm{x}}\right)+\tan^{-1}\left(\frac{3+2\log \mathrm{x}}{1-3(2\log \mathrm{x})}\right)$$

$$= \tan^{-1}(1)-\tan^{-1}(2\log \mathrm{x})+\tan^{-1}(3)+\tan^{-1}(2\log \mathrm{x})$$

$$\Rightarrow \mathrm{f(x)} = \tan^{-1}(1)+\tan^{-1}(3)$$

$$\Rightarrow \mathrm{f}'(\mathrm{x}) = 0$$

$$\Rightarrow \mathrm{f}'(5) = 0$$

83 **(2)**

Given, $\lim_{\alpha\to 0}\left[\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right] = -\frac{e}{2}$

$$\Rightarrow \lim_{\alpha\to 0}\frac{e\{e^{\cos(\alpha^n)-1}-1}{\cos(\alpha^n)-1}\cdot\frac{\cos(\alpha^n)-1}{\alpha^m} = \frac{-e}{2}$$

$$\Rightarrow \lim_{\alpha\to 0}e\left\{\frac{e^{\cos(\alpha^n)-1}-1}{\cos(\alpha^n)-1}\right\}\cdot\lim_{\alpha\to 0}\frac{-2\sin^2\frac{\alpha^n}{2}}{\alpha^m} = -e/2$$

$$\Rightarrow e\times 1\times(-2)\lim_{\alpha\to 0}\frac{\sin^2\left(\frac{\alpha^n}{2}\right)}{\frac{\alpha^{2n}}{4}}\cdot\frac{\alpha^{2n}}{4\alpha^m} = -\frac{e}{2}$$

$$\Rightarrow \quad e \times 1 \times -2 \times 1 \times \lim_{\alpha \to 0} \frac{\alpha^{2n-m}}{4} = -\frac{e}{2}$$

For this to be exists, $2n - m = 0$

$$\Rightarrow \quad \frac{m}{n} = 2$$

84 **(0)**

$$\lim_{x \to 0^+} f\left(g\left(h(x)\right)\right) = f\left(g(0^+)\right) = f(1^+) = 0$$

$$\lim_{x \to 0^-} f\left(g\left(h(x)\right)\right) = f\left(g(0^+)\right) = f(1^+) = 0$$

Hence $\lim_{x \to 0} f\left(g\left(h(x)\right)\right) = 0$

85 **(1)**

$$L = \lim_{x \to 0}\left[\frac{(1+x)^{\frac{1}{x}} - e + \frac{1}{2}ex}{x^2}\right]$$

$$= \lim_{x \to 0}\left[\frac{e^{\frac{1}{x}\log(1+x)} - e + \frac{1}{2}ex}{x^2}\right]$$

$$= \lim_{x \to 0}\left[\frac{e^{\frac{1}{x}\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots\right)} - e + \frac{1}{2}ex}{x^2}\right]$$

$$= \lim_{x \to 0}\left[\frac{e^{1 - \frac{x^2}{2} + \frac{x^3}{3} - \cdots} - e + \frac{1}{2}ex}{x^2}\right]$$

$$= \lim_{x \to 0}\left[\frac{e\left(e^{-\frac{x}{2} + \frac{x^3}{3} - \cdots}\right) - e + \frac{1}{2}ex}{x^2}\right]$$

$$= \lim_{x \to 0}\left[\frac{e\left\{1 + \left(-\frac{x}{2} + \frac{x^3}{3} - \cdots\right) + \frac{1}{2!}\left(-\frac{x}{2} + \frac{x^3}{3} - \cdots\right)^2 \cdots}{x^2}\right.$$

$$= \lim_{x \to 0}\left[\frac{e - \frac{ex}{2} + \left(\frac{1}{3} + \frac{1}{8}\right)ex^2 + ex^3\left(-\frac{1}{4} - \frac{1}{6}\right) + \cdots - e \cdots}{x^2}\right.$$

$$= \left(\frac{1}{3} + \frac{1}{8}\right)e = \frac{11e}{24} \cong \frac{29.9}{24}$$

$$\Longrightarrow [L] = \left[\frac{29.9}{24}\right] = 1$$